Subject Name: DIGITAL SIGNAL PROCESSING

Prepared by (Faculty (s) Name): Mrs. K.VANISREE

Year and Sem, Department: III Year II Sem ECE Dept.

Course Objectives:

This course is an essential course that provides design techniques for processing all type of signals in various fields. The main objectives are:

To provide background and fundamental material for the analysis and processing of digital signals.

To familiarize the relationships between continuous-time and discrete time signals and systems.

To study fundamentals of time, frequency and Z-plane analysis and to discuss the inter-relationships of these analytic method.

To study the designs and structures of digital (IIR and FIR) filters from analysis to synthesis for a given specifications.

The impetus is to introduce a few real-world signal processing applications.

To acquaint in FFT algorithms, Multi-rate signal processing techniques and finite word length effects.

Course Outcomes:

On completion of this subject, the student should be able to:

Perform time, frequency, and Z -transform analysis on signals and systems

Understand the inter-relationship between DFT and various transforms.

Understand the significance of various filter structures and effects of round off errors.

Design a digital filter for a given specification.

Understand the fast computation of DFT and appreciate the FFT processing.

Understand the tradeoffs between normal and multi rate DSP techniques and finite length word effects.

Important points / Definitions:

1. A signal is defined as any physical quantity that changes with time, distance, speed, position, pressure, temperature or some other quantity.

2. A System is a physical device that performs an operations or processing on a signal. Ex Filter or Amplifier.

3. DSP (Digital signal Processing) : If the input signal given to the system is digital then system does digital signal processing.

4. A two dimensional recursive filter is characterized by the difference equation which relates the input and output as,

$$y(m,n)=\sum_{k,l}\sum_{\in\alpha} a(k,l)x(m-k,n-l)-\sum_{k,l}\sum_{\in\beta-(0,0)} d(k,l)y(m-k,n-l)$$

5. The impulse response of the system represented by

$$h(m,n)=a(m,n)-\sum_{k,l}\sum_{\in\beta-(0,0)} d(k,l)h(m-k,n-l)$$

6.

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DIGITAL SIGNAL PROCESSING

B.Tech. III Year II Sem. L T P C Course Code: EC603PC 4 0 0 4

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UNIT - I Introduction: Introduction to Digital Signal Processing: Discrete Time Signals & Sequences, conversion of continuous to discrete signal, Normalized Frequency, Linear Shift Invariant Systems, Stability, and Causality, linear differential equation to difference equation, Linear Constant Coefficient Difference Equations, Frequency Domain Representation of Discrete Time Signals and Systems Realization of Digital Filters: Applications of Z – Transforms, Solution of Difference Equations of Digital Filters, System Function, Stability Criterion, Frequency Response of Stable Systems, Realization of Digital Filters – Direct, Canonic, Cascade and Parallel Forms.

UNIT - II Discrete Fourier Transforms: Properties of DFT, Linear Convolution of Sequences using DFT, Computation of DFT: Over-Lap Add Method, Over-Lap Save Method, Relation between DTFT, DFS, DFT and Z-Transform. All JNTU World
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Fast Fourier Transforms: Fast Fourier Transforms (FFT) - Radix-2 Decimation-in-Time and Decimation-in-Frequency FFT Algorithms, Inverse FFT, and FFT with General Radix-N.

UNIT - III IIR Digital Filters: Analog filter approximations – Butterworth and Chebyshev, Design of IIR Digital Filters from Analog Filters, Step and Impulse Invariant Techniques, Bilinear Transformation Method, Spectral Transformations.

UNIT - IV FIR Digital Filters: Characteristics of FIR Digital Filters, Frequency Response, Design of FIR Filters: Fourier Method, Digital Filters using Window Techniques, Frequency Sampling Technique, Comparison of IIR & FIR filters.

UNIT - V Multirate Digital Signal Processing: Introduction, Down Sampling, Decimation, Upsampling, Interpolation, Sampling Rate Conversion, Conversion of Band Pass Signals, Concept of Resampling, Applications of Multi Rate Signal Processing. Finite Word Length Effects: Limit cycles, Overflow Oscillations, Round-off Noise in IIR Digital Filters, Computational Output Round off Noise, Methods to Prevent Overflow, Trade off between Round Off and Overflow Noise, Measurement of Coefficient Quantization Effects through Pole-Zero Movement, Dead Band Effects.

TEXT BOOKS: 1. Digital Signal Processing, Principles, Algorithms, and Applications: John G. Proakis, Dimitris G. Manolakis, Pearson Education / PHI, 2007. 2. Discrete Time Signal Processing – A. V. Oppenheim and R.W. Schaffer, PHI, 2009 3. Fundamentals of Digital Signal Processing – Loney Ludeman, John Wiley, 2009

REFERENCES: 1. Digital Signal Processing – Fundamentals and Applications – Li Tan, Elsevier, 2008 2. Fundamentals of Digital Signal Processing using MATLAB – Robert J. Schilling, Sandra L. Harris, Thomson, 2007 3. Digital Signal Processing - A Practical approach, Emmanuel C. Ifeachor and Barrie W. Jervis, 2nd Edition, Pearson Education, 2009

Unit-I Introduction:

Short Questions (minimum 10 previous JNTUH Questions – Year to be mentioned)
1. Discuss the stability criterion for digital filter. [Nov/Dec 2016]

Assume that the characteristic equation is as follows, $P(z) = a_0z^n + a_1z^{n-1} + ... + a_{n-1}z + a_n$ where $a_0 > 0$.

1. $|a_n| < a_0$

2. $P(z)|_{z=1} > 0$

3. $P(z)|_{z=-1} > 0$ for n even and $P(z)|_{z=-1} < 0$ for n odd

4. $|b_{n-1}| > |b_0|$ $|c_{n-2}| > |c_0|$ . ... $|q_2| > |q_0|$

Example : The characteristic equation: P(z) = z 4 − 1.2z 3 + 0.07z 2 + 0.3z − 0.08 = 0 Thus, a0 = 1 a1 = −1.2 a2 = 0.07 a3 = 0.3 a4 = −0.08 We will now check the stability conditions.

1. |an| = |a4| = 0.08 < a0 = 1 ⇒ First condition is satisfied.

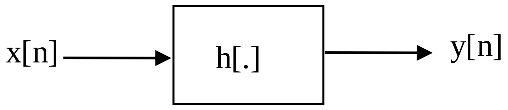
2. P(1) = 1 − 1.2 + 0.07 + 0.3 − 0.08 = 0.09 > 0 ⇒ Second condition is satisfied.

3. P(−1) = 1 + 1.2 + 0.07 − 0.3 − 0.08 = 1.89 > 0 ⇒ Third condition is satisfied. Jury Table b3 = an a0 a0 an = 0.0064 − 1 = −0.9936 b2 = an a1 a0 a3

= −0.08 × 0.3 + 1.2 = 1.176 Rest of the elements are also calculated in a similar fashion. The elements are b1 = −0.0756 b0 = −0.204 c2 = 0.946 c1 = −1.184 c0 = 0.315. One can see |b3| = 0.9936 > |b0| = 0.204 |c2| = 0.946 > |c0| = 0.315 All criteria are satisfied. Thus the system is stable

2. Explain the frequency representation of discrete time systems [Nov/Dec 2016]

Let us assume we have an LTI system



If $x[n] = e^{j2\pi f_d n}$ **then**

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=-\infty}^{\infty} h[k]e^{j2\pi f_d (n-k)}$$

$$= e^{j2\pi f_d n} \sum_{k=-\infty}^{\infty} h[k]e^{-j2\pi f_d k}$$

$$= \underbrace{e^{j2\pi f_d n}}_{} \underbrace{H(e^{jw})}_{}$$

Eigenfunction eigenvalue

Example:

Let $x[n] = A\cos(2\pi fn + \Phi) = \frac{A}{2}\left\{e^{j(\Phi + 2\pi fn)} + e^{-j(\Phi + 2\pi fn)}\right\}$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

➔ $$= \frac{A}{2}\left\{\sum_{k=-\infty}^{\infty} h[k]e^{j(\Phi + 2\pi f(n-k))} + \sum_{k=-\infty}^{\infty} h[k]e^{-j(\Phi + 2\pi f(n-k))}\right\}$$

$$= \frac{A}{2}\left\{H\left(e^{jw}\right)e^{j(\Phi + 2\pi fn)} + H\left(e^{-jw}\right)e^{-j(\Phi + 2\pi fn)}\right\}$$

A special case of this problem exist when h[n] is real

$$H\left(e^{-jw}\right) = \overline{H\left(e^{jw}\right)}$$

In this case

$$y[n] = A\left|H\left(e^{jw}\right)\right|\cos(2\pi fn + \Phi + \Theta) \qquad where \quad \Theta = angH\left(e^{jw}\right)$$

3. Write four advantages of Digital Signal Processing over Analog Signal Processing [May 2016]

1. A digital programmable system allows flexibility in reconfiguring the digital signal processing operations by changing the program. In analog redesign of hardware is required.
2. In digital accuracy depends on word length, floating Vs fixed point arithmetic etc. In analog depends on components.
3. Can be stored on disk.
4. It is very difficult to perform precise mathematical operations on signals in analog form but these operations can be routinely implemented on a digital computer using software.

4. Show that the frequency response of a discrete system is a periodic function of frequency [May 2016]

A discrete-time sequence is periodic with a period of N samples if
x[n] = x[n + kN] for all integer values of k. Note that N has to be a positive integer. If a sequence is not periodic, it is aperiodic or non-periodic.
We know that continuous-time sinusoids are periodic. For instance, the continuous-time signal $x(t) = \cos(\omega_0 t)$ has a frequency of ω0 radians per second or $f_0 = \omega_0/2\pi$ Hz. The period of this sinusoidal signal is T = 1/f0 seconds.
Now consider a discrete-time sequence x [n] based on a sinusoid with angular frequency ω_0: $x[n] = \cos(\omega_0 n)$
If this sequence is periodic with a period of N samples, then the following must be true: $\cos \omega_0 (n + N) = \cos \omega_0 n$
However, the left hand side can be expressed as $\cos \omega_0 (n + N) = \cos(\omega_0 n + \omega_0 N)$
and the cosine function is periodic with a period of 2π and therefore the right hand side of is given by $\cos \omega_0 n = \cos(\omega_0 n + 2\pi r)$
$\omega_0 N = 2\pi r$
$\Rightarrow 2\pi f0 N = 2\pi r$
$\Rightarrow f0 = r N$
where $\omega_0 = 2\pi f_0$. Since both r and N are integers, a discrete-time sinusoidal sequence is periodic if its frequency is a rational number. Otherwise, it is non-periodic.

5. What is an LTI system? [May 2017]

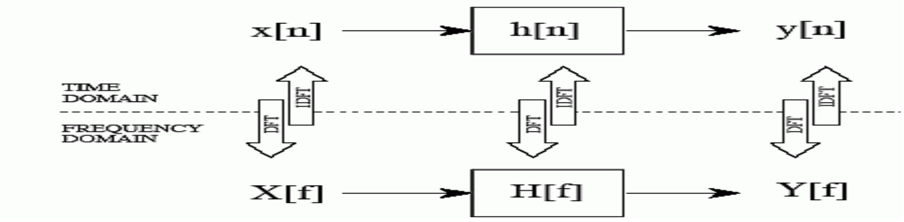
Linear time invariant (LTI) system is "the system which obeys the linear property and time invariant property".
LTI system obeys superposition principle. Therefore, it obeys linear property. And also the LTI system will not vary with respect to time.
Consider the input signals $x_1(t)$ and $x_2(t)$ and corresponding output signals are $y_1(t)$ and $y_2(t)$,
Consider the constants *a* and *b*

$$T\left(ax_1(t)+bx_2(t)\right)=aTx_1(t)+bTx_2(t)$$
$$=ay_1(t)+by_2(t)$$

Consider the signal $x(t)$ gives the output $y(t)$
If input signal is varied with respect to time as $x(t+t_1)$ then output is also varied as $y(t+t_1)$
These signals are linear and time invariant signals.

6. Define the frequency response of a discrete-time system [May 2017]

The DFT can be used to represent every output signal in a similar form. This means that any linear system can be completely described by how it changes the amplitude and phase of cosine waves passing through it. This information is called the system's frequency response. Since both the impulse response and the frequency response contain complete information about the system, there must be a one-to-one correspondence between the two. Given one, you can calculate the other. The relationship between the impulse response and the frequency response is one of the foundations of signal processing: A system's frequency response is the Fourier Transform of its impulse response.



7. Define stability. **[Dec 2017]**

A stable system satisfies the BIBO bounded input for bounded output input for bounded out put condition. Here, bounded means finite in amplitude. For a stable system, output should be bounded or finite, for finite or bounded input, at every instant of time.

Some examples of bounded inputs are functions of sine, cosine, DC, signum and unit step.

8. List the applications of Z- transform **[Dec 2017]**

- Uses to analysis of digital filters.
  Used to simulate the continuous systems.
  Analyze the linear discrete system.
  Used to finding frequency response.
  Analysis of discrete signal.
  Helps in system design and analysis and also checks the systems stability.
  For automatic controls in telecommunication.
  Enhance the electrical and mechanical energy to provide dynamic nature of the system

9. Show that δ(n) = u(n) – u(n-1) **[April 2018]**

The unit step sequence is defined as u[n] = 1, n ≥ 0, 0, n< 0. (2.6) The unit step is related to the unit impulse by u[n] = n k=−∞ δ[k]
that is, the value of the unit step sequence at (time) index n is equal to the accumulated sum of the value at index n and all previous values of the impulse sequence.
An alternative representation of the unit step in terms of the impulse is obtained by interpreting the unit step in terms of a sum of delayed impulses,
In this case, the nonzero values are all unity, so u[n] = δ[n] + δ[n − 1] + δ[n − 2]+··· (2.8a) or u[n] = ∞ k=0 δ[n − k]. As yet another alternative, the impulse sequence can be expressed as the first backward difference of the unit step sequence, i.e., δ[n] = u[n] − u[n − 1].

10. Find the Z-transform $f(n) = n^2(n)$ **[April 2018]**

$$f(n) = a^n u(n) + a^{-n} u(-n)$$

$$Z(f(n)) = \frac{Z}{Z-a} + \frac{Z^{-1}}{Z^{-1}-a}$$

11. What are the properties of frequency respons H(e^{jw}) of an LTI system? [Dec 2018]

12. What is the relation between Z-transform and DTFT? [Dec 2018]

i. One can obtain the DTFT from the z-transform X(z) by as follows:

$X(z)|z=ejw=X(\omega)$

In other words, if you restrict the z-transoform to the unit circle in the complex plane, then you get the Fourier transform (DTFT).

ii. One can also obtain the Z-Transform from the DTFT.

Write the z-transform $X(z)=X(rejw)$ using polar coordinates for the complex number z. Then

$X(z)=\sum-\infty\infty x[n]z-n=\sum-\infty\infty x[n](rejw)-n=\sum-\infty\infty x[n]r-ne-jwn=F(x[n]r-n)$

So the z-transform is like a DTFT after multiplying the signal by the signal $y[n]=r-n$.

Long Questions (minimum 10 previous JNTUH Questions – Year to be mentioned)

1. Explain the canonical form of digital filter realization. [Nov/Dec 2016]

2. Discuss the concept of stability and causality with examples [Nov/Dec 2016]

The system is said to be stable only when the output is bounded for bounded input. For a bounded input, if the output is unbounded in the system then it is said to be unstable.

Note: For a bounded signal, amplitude is finite.

Example : y (t) = x^2(t)

Let the input is u(t) (unit step bounded input) then the output y(t) = u2(t) = u(t) = bounded output.

Hence, the system is stable.

A system is said to be causal if its output depends upon present and past inputs, and does not depend upon future input.

For non causal system, the output depends upon future inputs also.

Example : y(n) = 2 x(t) + 3 x(t-3)

For present value t=1, the system output is y(1) = 2x(1) + 3x(-2).

Here, the system output only depends upon present and past inputs. Hence, the system is causal.

3. Test the following systems for linearity, time invariance, causality and stability. y(n) = sin(2nfπ/F)x(n)
[May 2016]

4. A digital system is characterized by the following difference equation: Y(n) = x(n)+ay(n-1) Assuming that the system is relaxed initially, determine its impulse response [May 2016]

5. Determine whether each of the following systems defined below is (i) Causal (ii) Linear (iii) Dynamic (iv) Time invariant (v) Stable.
[May 2017]

6. If X(n) is a casual sequence , find the z- transform of the following sequences
(i) x(n) = nu(n) (ii) x(n) = n u (n-1)
b. find the sequence of y(n) + y(n+1)-2y(n-2) = u(n-1) + 2u (n-2) due to v(-1) = 0.5 v(-2) = 0.25 [May 2017]

7.

.a) Check whether the following systems are stable, causal.

(i) $h(t) = te^{at}\ u(t)$ (ii) $h(n) = e^{n/2}\ u(n-4)$

b) Determine the impulse response of the system described by the difference equation y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1) using Z transform.

[May 2017]

8.

.a) A system is described by the difference equation y(n)-y(n-1)-y(n-2) = x(n-1). Assuming that the system is initially relaxed, determine its unit sample response h(n).

b) Show that an LSI system can be described by its unit step response.

[May 2017]

9. An LTI system is characterized by an impulse response $h(n) = \left(\frac{3}{4}\right)^n u(n)$ **Find the step response of the system. Also, evaluate the output of the system at n=±5.**

b) Consider a discrete-time system characterized by the following input-output relationship $y\ n = x\ n - 2 - 2(n - 17)$. Determine whether the system is memory less, time-Invariant, linear, causal and stable. [April 2018]

10. Given the difference equation $y\ n + b2y\ n - 2 = 0$ for $n \geq 0$ and < 1. With initial conditions $yf\ -1 = 0\ and\ y\ -2 = -1$, Show that $y(n) = b^{n+2}\cos\left(\frac{n\pi}{2}\right)$

b) Find the Z-transform of the sequence f(n) defined below: [April 2018]

11. Determine the stability for the following systems and test the causality

$h(n) = u(n); h(n) = 4^n u(2-n); h(n) = 2^n u(n); h(n) = e^{-6}|n|; h(n) = 5^n u(3-n)$

For each impulse response listed below, determine whether the corresponding system is (i) causal (ii) stable. H(n) = δ(n) +sinπn [Dec 2018]

12, Find the z-transform and ROC of the following sequence

$(i)\ (-1)^n \cos\left(\frac{\pi}{3} n\right) u(n) \quad (ii)\ x(n) = (0.25)^n u(n) + (0.5)^n u(n)$

b. Determine H(z) for the given systems. Discuss stability, and if possible determine H(ejw) from H(z). [Dec 2018]

Fill in the Blanks / Choose the Best: (Minimum 10 to 15 with Answers)

1. If x(n) is a discrete-time signal, then the value of x(n) at non integer value of 'n' is:
a) Zero
b) Positive
c) Negative
d) Not defined **[d]**

2. The discrete time function defined as u(n)=n for n=0;=0 for n<0 is an:
a) Unit sample signal
b) Unit step signal
c) Unit ramp signal
d) None of the mentioned **[c]**

3. The even part of a signal x(t) is:
a) x(t)+x(-t) b) x(t)-x(-t)
c) (1/2)*(x(t)+x(-t)) d) (1/2)*(x(t)-x(-t)) **[c]**

4.For a continuous time signal x(t) to be periodic with a period T, then x(t+mT) should be equal to:
a) x(-t) b) x(mT)
c) x(mt) d) x(t) **[d]**

5. Let x1(t) and x2(t) be periodic signals with fundamental periods T1 and T2 respectively. Which of the following must be a rational number for x(t)=x1(t)+x2(t) to be periodic?
a) T1+T2 b) T1-T2
c) T1/T2 d) T1*T2 **[c]**

6. What is the configuration of system for digital processing of an analog signal?
a) Analog signal|| Pre-filter ->D/A Converter -> Digital Processor -> A/D Converter -> Post-filter
b) Analog signal|| Pre-filter ->A/D Converter -> Digital Processor -> D/A Converter -> Post-filter
c) Analog signal|| Post-filter ->D/A Converter -> Digital Processor -> A/D Converter -> Pre-filter
d) None of the mentioned **[b]**

7. A system is said to be unstable if
a. None of the poles of its transfer function is shifted to the right half of s-plane
b. At least one zero of its transfer function is shifted to the right half of s-plane
c. At least one pole of its transfer function is shifted to the right half of s-plane
d. At least one pole of its transfer function is shifted to the left half of s-plane **[b]**

8. A system is said to be marginally unstable if
a. None of its zeros of its transfer function lies on the jω axis of s-plane
b. At least one zero of its transfer function lies on the jω axis of s-plane
c. None of its poles of its transfer function lies on the jω axis of s-plane
d. At least one pole of its transfer function lies on the jω axis of s-plane **[d]**

9. For a linear phase filter, if Z_1 is zero then what would be the value of Z_1^{-1} or 1/ Z_1?
a. Zero **b.** Unity
c. Infinity **d.** Unpredictable **[a]**

10. In linear phase realization, equal valued coefficients are taken common for reducing the requisite number of
a. adders **b.** subtractors
c. multipliers **d.** dividers **[c]**

11. x(t) or x(n) is defined to be an energy signal, if and only if the total energy content of the signal is a__________________ [Finite quantity]

12. What is the z-transform of the signal x(n)=[3(2n)-4(3n)]u(n)? [3/(1-2z-1)-4/(1-3z-1)]

13. According to Time shifting property of z-transform, if X(z) is the z-transform of x(n) then what is the z-transform of x(n-k)?____________ [z-kX(z)]

14. If x(n) is a stable sequence so that X(z) converges on to a unit circle, then the complex cepstrum signal is defined as: __________ [X^{-1}(ln X(z))]
15. The phase associated with the frequency component of discrete-time Fourier series(DTFS)? [$e^{j2\pi kn/N}$]

16. What is the energy of a discrete time signal in terms of X(ω)? [$\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\omega)|^2 \, d\omega$]
17. Time shifting of discrete time signal means [y[n] = x[n-k]]
18. Causal systems are the systems in which
[The output of the system depends on the present and the past inputs]
19. Circular shift of an N point is equivalent to
[Linear shift of its periodic extension and its vice versa]
20. Parallel form of realisation is done in_____ [High speed filtering applications]

Unit-II

Important points / Definitions:
1 the **discrete Fourier transform** (**DFT**) converts a finite sequence of equally-spaced samples of a function into a same-length sequence of equally-spaced samples of the discrete-time Fourier transform (DTFT), which is a complex-valued function of frequency.

2.A **twiddle factor**, in fast Fourier transform (FFT) algorithms, is any of the trigonometric constant coefficients that are multiplied by the data in the course of the algorithm.

3. A fast **Fourier transform** (**FFT**) is an algorithm that computes the discrete **Fourier transform** (DFT) of a sequence, or its inverse (IDFT). Fourier analysis converts a signal from its original domain (often time or space) to a representation in the frequency domain and vice versa.

4. **FFT** is based on divide and conquer algorithm where you divide the signal into two smaller signals, compute the **DFT** of the two smaller signals and join them to get the **DFT** of the larger signal. The order of complexity of **DFT** is O(n^2) while that of **FFT** is O(n.logn) hence, **FFT** is **faster than DFT.**

5. The DFT is defined as such: $X[k] = \sum_{n=0}^{N-1} x[n] e^{\frac{-j2\pi kn}{N}}$

6. The original sequence {x(n)} can be retrieved by the inverse discrete Fourier transform (IDFT)

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, W_N^{-kn}, \qquad n = 0, \ldots, N-1 .$$

7.

ZERO PADDING

If $x(n)$ has length N and we want to evaluate

$$X_N(f) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi fn}$$

at $M > N$ frequency values, calculate the M-point DFT of

$$x_{ZP}(n) = \{x(0), x(1), \ldots, x(N-1), \underbrace{0, \ldots, 0}_{M-N \text{ zeros}}\}$$

8..

FFT (RADIX-2) OBSERVATION

- Length N sequence $x(n)$, $X(k) = \text{FFT}_N[x(n)]$
  - even elements: $x_e(m) = x(2m)$, $X_e(k) = \text{FFT}_{N/2}[x_e(m)]$
  - odd elements: $x_o(m) = x(2m+1)$, $X_o(k) = \text{FFT}_{N/2}[x_o(m)]$

$$\Rightarrow X(k) = X_e(k) + e^{-j2\pi \frac{k}{N}} X_o(k)$$

Parseval's theorem

9. $$\sum_{n=0}^{N-1} x(n) y^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_k Y^*(k)$$

Short Questions (minimum 10 previous JNTUH Questions – Year to be mentioned)

1. Write any two properties of DFS. [Nov/Dec 2016] [Dec 2017]

The following properties should be noted.

	Sequence	DFS
1	$x_p(n+m)$	$W_N^{-km} X_p(k)$
2	$x_p^*(n)$	$X_p^*(-k)$
3	$x_p^*(-n)$	$X_p^*(k)$

	Sequence	DFS
4	Re $[x_p(n)]$	$X_{pe}(k)$
5	j Im $[x_p(n)]$	$X_{po}(k)$

2. Differentiate between Decimation-in-time and Decimation-in-frequency [Nov/Dec 2016]

DIT radix – 2 FFT	DIF radix – 2 FFT
The time domain sequence is decimated.	The frequency domain sequence is decimated.
When the input is in bit reversed order, the output will be in normal order and vice versa.	When the input is in bit normal order, the output will be in bit reversed order and vice versa.
In each stage of computations, the phase factors are multiplied before add and subtract operations.	In each stage of computations, the phase factors are multiplied after add and subtract operations.
The value of N should be expressed such that $N = 2^m$ and this algorithm consists of m stages of computations.	The value of N should be expressed such that $N = 2^m$ and this algorithm consists of m stages of computations.
Total number of arithmetic operations is NlogN complex additions and (N/2) logN complex multiplications.	Total number of arithmetic operations is NlogN complex additions and (N/2) logN complex multiplications.

3. Give the relation between DTFT and Z-Transform. [May-2016]

The DFT and *z*-transform. The *z*-transform of a discrete time sequence of finite duration is given by

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} \qquad (11.89)$$

Let us consider a finite duration sequence $x(n)$, $0 \le n \le N-1$.The above equation reduces to

$$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n} \qquad (11.90)$$

DFT of $x(n)$ is given by

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}} \qquad (11.91)$$

4. Distinguish between Linear convolution and circular convolution [May-2016] [Dec 2018]

6. Distinguish between linear convolution and circular convolution. [Dec 2009]

S. No	Linear Convolution	Circular Convolution
1.	If x(n) is a sequence of L number of samples and h(n) with m number of samples, after convolution y(n) will contain N = L + M – 1 samples.	If x(n) is a sequence of L number of samples and h(n) with m number of samples, after convolution y(n) will contain N = Max(L,M) samples
2.	Linear convolution can be used to find the response of a linear filter.	Circular convolution can be used to find the response of a linear filter
3.	Zero padding is not necessary to find the response of a linear filter.	Zero padding is necessary to find the response of a linear filter.

- Let us consider two sequences
  x(n)={1, 2, 3} and
  h(n) = {1, 2, 3, 4, 5}
  Now x(n) has **3** samples
  and h(n) has **5** samples.
- To compute circular convolution x(n) must also have **5** samples.
  This is done by **zero padding.**
- Now x(n) = {1,2,3,0,0} and h(n) = {1, 2, 3, 4, 5}so, circular convolution will also have 5 samples in its output.
- To compute the linear convolution , N = L + M - 1
  = 3 + 5 - 1
  = 7
- Hence linear convolution will have 7 samples in its output.

5. Define discrete Fourier series. [May 2017]

A Fourier series is a representation of a function in terms of a summation of an infinite number of harmonically-related sinusoids with different amplitudes and phases. The amplitude and phase of a sinusoid can be combined into a single complex number, called a Fourier *coefficient*. The Fourier series is a periodic function. So it cannot represent any arbitrary function. It can represent either**:**

(a) a periodic function, or

(b) a function that is defined only over a finite-length interval; the values produced by the Fourier series outside the finite interval are irrelevant

6.Obtain the circular convolution of the sequence x(n)={1,2,1}; h(n)={1,-2,2}. [May 2017]

7.What is the value of x(n)*h(n), 0≤n≤11 for the sequences x(n)={1,2,0,-3,4,2,-1,1,2,3,2,1,-3} and h(n)={1,1,1} if we perform using overlap save fast convolution technique? [Dec 2017]

8. State and prove the any three properties of DFT. [April 2018]

Linearity

Circular shift of a sequence: if $X(k) = \mathcal{DFT}\{x(n)\}$ then

$$X(k)e^{-j2\pi\frac{km}{N}} = \mathcal{DFT}\{x((n-m) \bmod N)\}$$

Also if $x(n) = \mathcal{DFT}^{-1}\{X(k)\}$ then

$$x((n-m) \bmod N) = \mathcal{DFT}^{-1}\{X(k)e^{-j2\pi\frac{km}{N}}\}$$

where the operation $\bmod N$ denotes the periodic extension $\tilde{x}(n)$ of the signal $x(n)$:

$$\tilde{x}(n) = x(n \bmod N).$$

DFT: Parseval's Theorem

$$\sum_{n=0}^{N-1} x(n)y^*(n) = \frac{1}{N}\sum_{k=0}^{N-1} \mathbf{X}(k)\mathbf{Y}^*(k)$$

Using the matrix formulation of the DFT, we obtain

$$\begin{aligned} \boldsymbol{y}^H\boldsymbol{x} &= \left(\frac{1}{N}W^H\mathbf{Y}\right)^H\left(\frac{1}{N}W^H\mathbf{Y}\right) \\ &= \frac{1}{N^2}\mathbf{Y}^H\underbrace{WW^H}_{NI}\mathbf{X} = \frac{1}{N}\mathbf{Y}^H\mathbf{X}. \end{aligned}$$

9. What is the basic operation of DIF algorithm? [April 2018]

DIF FFT

1. DIFFFT algorithms are based upon decomposition of the output sequence into smaller and smaller sub sequences.

2. In this output sequence X(k) is considered to be splitted into even and odd numbered samples

3. Splitting operation is done on frequency domain sequence.

4. In DIFFFT, input sequence is in natural order. And DFT should be read in bit reversed order.

10. What is zero padding? What are its uses? [Dec 2018]

.In order to increase the resolution the record length TR should be increased. Once the sampling frequency is fixed, to increase the resolution one has to increase N. This is achieved by 'Zero padding'.

Let the DTFT of the finite-duration signal xN(n)={xN(0),xN(1), xN(2),……,xN(N-1)} , whose record length is $T_R = N\ T_S$ or simply N, is given as:

$$X_N(e^{j\omega}) = \sum_{n=0}^{N-1} x_N(n) e^{-j\omega n}$$

By padding L-N zeros to xN(n) we can artificially increase the record length to $T_R = L\ T_S$ or simply L. The DTFT of the zero padded L-length sequence

$$x_L(n) = \{x_N(0), x_N(1),\ x_N(2), \ldots\ldots, x_N(N-1), \underbrace{0, 0, \ldots\ldots\ldots\ldots\ldots, 0}_{L-N\ zeros}\}$$

Long Questions (minimum 10 previous JNTUH Questions – Year to be mentioned)

1. Explain the properties of DFT. [Nov/Dec 2016] [Dec 2018]

2. Explain Radix- 2 Decimation- in-Time algorithms [Nov/Dec 2016]

3. By taking an example compute DFT by using Over-Lap save method.[May 2016]

4. Compute the circular convolution of the sequences x1(n) = {1, 2, 0, 1} and x2(n) = {2, 2, 1, 1} Using DFT approach. b) What is FFT? Calculate the number of multiplications needed in the calculation of DFT using FFT algorithm with 32 point sequence
[May 2016]

5.

Design a chebyshev filter for the following specifications using (a) bilinear transformation. (b) Impulse Invariance method.

$$0.8 \le \left|H\left(e^{jw}\right)\right| \le 1 \quad 0 \le w \le 0.2\pi$$
$$\left|H\left(e^{jw}\right)\right| \le 0.2 \quad 0.6\pi \le w \le \pi$$

[May 2017]

6

Design a lowpass filter that will operate on the sampled analog data such that the cutoff frequency is 200Hz and at 400Hz, the attenuation is atleast 20dB with a monotonic shape past 200Hz. Take T = $\frac{1}{2000}$ secs and use normalized lowpass filter.

A third-order Butterworth low pass filter has the transfer function:

$H(s) = \frac{1}{(s+1)(s^2+s+1)}$. Design H(z) using Impulse Invariance method.

[May 2017]

7.Implement the Decimation in frequency FFT algorithm of N-point DFT where N-Also explain the steps involved in this algorithm [Dec 2017]

8. If x(n) is a periodic sequence with a period N, also periodic with period 2N. X1(K) denotes the discrete Fourier series coefficient of x(n) with period N and X2(k) denote the discrete Fourier series coefficient of x(n) with period 2N. Determine X2(K) in terms of X1(K).
b) What is FFT? Calculate the number of multiplications needed in the calculation of DFT using FFT algorithm with 32 point sequence. [Dec 2017]

9. Find the IDFT of the sequence X(K) given below X(K) = {1,0,0, j, 0, -j, 0,0}
b) Obtain the 10 point DFT of the sequence $x\ n = \delta\ n + 2(n-5)$. [April 2018]

10.Find the IDFT of the sequence X(K) = {20, -5.828-j2.414,0,-0.712-j0.414,0,-0.172+j0.414,0,-5.828+j2.414} using DIT- FFT algorithm.
b) Using FFT and IFFT, determine the output of system if input x(n)={2,2,4} and impulse response h(n) = {1,1}. [April 2018]
11.Find 8-point DFT of the sequence x(n) = cos(nπ/4) . [Dec 2018]

12. Given x(n) = 2^n and N=8, find X(k) using DIT-FFT algorithm.
b) Find the 8-point DFT of the given sequence x(n) = {10,2,3,4,5,6,7}. [Dec 2018]

Fill in the Blanks / Choose the Best: (Minimum 10 to 15 with Answers)

1. Discrete cosine transforms (DCTs) express a function or a signal in terms of

a. Sum of cosine functions oscillating at different frequencies
b. Sum of cosine functions oscillating at same frequencies

c. Sum of cosine functions at different sampling intervals
d. Sum of cosine functions oscillating at same sampling intervals **[a]**

2. In Overlap-Add Method with linear convolution of a discrete-time signal of length L and a discrete-time signal of length M, for a length N, zero padding should be of length

a.L,M>N
b.L,M=N
c.L,M<N
d. L, M < N^2 **[c]**

3. Overlap-Add Method Deals with principles that

a. The linear convolution of a discrete-time signal of length L and a discrete-time signal of length M produces a discrete-time convolved result of length L + M – 1

b. The linear convolution of a discrete-time signal of length L and a discrete-time signal of length M produces a discrete-time convolved result of length L + M

c. The linear convolution of a discrete-time signal of length L and a discrete-time signal of length M produces a discrete-time convolved result of length 2L + M – 1

d. The linear convolution of a discrete-time signal of length L and a discrete-time signal of length M produces a discrete-time convolved result of length 2L + 2M – 1 **[c]**

4. The overlap save method is used to calculate

a. The discrete convolution between a sampled signal and a finite impulse response (FIR) filter
b. The discrete convolution between a sampled signal and an infinite impulse response (IIR) filter
c. The discrete convolution between a very long signal and a finite impulse response (FIR) filter
d. The discrete convolution between a very long signal and a infinite impulse response (IIR) filter **[c]**

5. Radix - 2 FFT algorithm performs the computation of DFT in

a. N/2Log2 N multiplications and 2Log2 N additions

b. N/2Log2 N multiplications and NLog2 N additions

c. Log2 N multiplications and N/2Log2 N additions

d. NLog2 N multiplications and N/2Log2 N additions **[b]**

6.. If x(n) is a finite duration sequence of length L, then the discrete Fourier transform X(k) of x(n) is given as:

a) $\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}$ (L<N)(k=0,1,2...N-1)

b) $\sum_{n=0}^{N-1} x(n)e^{j2\pi kn/N}$ (L<N)(k=0,1,2...N-1)

c) $\sum_{n=0}^{N-1} x(n)e^{j2\pi kn/N}$ (L>N)(k=0,1,2...N-1)

d) $\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}$ (L>N)(k=0,1,2...N-1) **[a]**

7. If X(k) discrete Fourier transform of x(n), then the inverse discrete Fourier transform of X(k) is:

a) $\frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{-j2\pi kn/N}$

b) $\sum_{k=0}^{N-1} X(k)e^{-j2\pi kn/N}$

c) $\sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}$

d) $\frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}$ **[d]**

8. A finite duration sequence of length L is given as x(n) =1 for 0≤n≤L-1=0 otherwise , then what is the N point DFT of this sequence for N=L?

a) X(k) =L for k=0, 1,2….L-1
b) X(k) =L for k=0
=0 for k=1,2….L-1
c) X(k) =L for k=0
=1 for k=1,2….L-1
d) None of the mentioned **[b]**

9. The Nth rot of unity W_N is given as:

a) $e^{j2\pi N}$
b) $e^{-j2\pi N}$
c) $e^{-j2\pi/N}$
d) $e^{j2\pi/N}$ **[c]**

10. What is the DFT of the four point sequence x(n)={0,1,2,3}?

a) {6,-2+2j-2,-2-2j}
b) {6,-2-2j,2,-2+2j}
c) {6,-2+2j,-2,-2-2j}
d) {6,-2-2j,-2,-2+2j} **[c]**

11. **The computational procedure for Decimation in frequency algorithm takes______________ stages** **[Log2 N stages]**

12. DIT algorithm divides the sequence into_______ [Even and Odd Samples]

13. FFT may be used to calculate_____________ [DFT and IDFT]

14. The Cooley–Tukey algorithm of FFT is a _____ [Divide and conquer algorithm]

15. For the calculation of N- point DFT, Radix -2 FFT algorithm repeats [N Log2 N)^2/2]

16. the N point DFT of a sequence whose Fourier series coefficients is given by c_k, then X(k) is _______________ [X(k)=Nc_k]

17. **What is the DFT of the four point sequence x(n)={0,1,2,3}? {6,-2+2j,-2,-2-2j}]**

18. If $W_4^{100}=W_x^{200}$, then what is the value of x? [8]

19. What is the z-transform of the finite duration signal **[$2z^2 + 4z + 5 + 7z^{-1} + z^{-3}$]**
$x(n)=\{2,4,\underset{\uparrow}{5},7,0,1\}$?

20. What is the ROC of z-transform of finite duration anti-causal sequence? [Entire z-plane, except at z=∞]